

# A PERFORMANCE CHARACTERISATION IN ADVANCED DATA SMOOTHING TECHNIQUES

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## Abstract

A comparison paper is presented to evaluate the results from five smoothing filters. The filters are linear, nonlinear isotropic and nonlinear anisotropic designed to smooth homogeneous areas while preserving the higher moments in the data. The methods are outlined and then evaluated on the extent to which edge information is preserved and unwanted noise is suppressed.

**Keywords:** *Smoothing, adaptive, Savitzky-Golay, diffusion*

## 1 Introduction

The noisy images presented in medical imaging has led researchers to investigate methods of smoothing image noise while maintaining important information such as edges.

There are two main types of smoothing, linear and non-linear. Both of these types have being extensively studied in literature. Traditional linear filters such as mean, average and Gaussian attempt to remove noise by replacing each pixel by an average or weighted average of its spatial neighbours [2]. While this reduces the amount of noise present in the image, it also has the disadvantage of removing or blurring the edges. Nonlinear filters, the most common being the median filter, modifies the value of the pixel by some nonlinear function of the pixel value and its spatial neighbours. Nonlinear filters maintain the edges but the filtering results in a loss of resolution by suppressing fine details. More recently, the use of edge-based diffusion has emerged [1, 3, 4, 5, 6]. These filters require a trade-off between smoothing efficiency, preservations of discontinuities and the generation of artifacts. In short the diffusion or smoothing term is a variable over space and time and in [3], this term is a function of the magnitude of the gradient intensity at the point in question. Gerig [4] extended this case to 3D and performed the diffusion on medical volumes. Perona and Malik's [3] diffusion has the disadvantage that it stopped the diffusion at edges, this was advanced by [7] by permitting diffusion along the direction of the edges making it anisotropic.

This paper compares five filters. The linear Savitzky-Golay filter is a convolution of the image with the least-squares fitting of a polynomial. The basic Gaussian filter is a convolution with a Gaussian mask, nonlinear adaptive filtering which filters the image but smooths less in areas of local discontinuities and high spatial variance. Nonlinear diffusion, which again smooths with an exponential with no smoothing occurring where the gradient has high values. Finally anisotropic Gaussian smoothing which uses a scaled and shaped Gaussian mask to smooth along the direction of high gradients and never across the gradients.

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## 2 Savitzky-Golay Filter

The Savitzky-Golay [8] smoothing filter was introduced for smoothing data and for computing the numerical derivatives. The smoothed points are found by replacing each data point with the value of its fitted polynomial. The process of Savitzky-Golay is to find the coefficients of the polynomial which are linear with respect to the data values. Therefore the problem is reduced to finding the coefficients for fictitious data and applying this linear filter over the complete data. The size of the smoothing window is given as  $N \times N$  where  $N$  is odd, and the order of the polynomial to fit is  $k$ , where  $N > k + 1$ .

$$f(x_i, y_i) = a_{00} + a_{10}x_i + a_{01}y_i + a_{20}x_i^2 + a_{11}x_iy_i + a_{02}y_i^2 + \dots + a_{0k}y_i^k \quad (1)$$

We then want to fit a polynomial of type in Eq. 1 to the data. Solving the least squares we can find the polynomial coefficients. We start with the general equation,  $A \cdot a = f$ , where  $a$  is the vector of polynomial coefficients  $a = (a_{00} \ a_{01} \ a_{10} \ \dots \ a_{0k})^T$ . We can then compute the coefficient matrix as follows,  $(A^T \cdot A) \cdot a = (A^T \cdot f)$ , which in least squares can be written as  $a = (A^T \cdot A)^{-1} \cdot (A^T \cdot f)$ .

Due to the linear-squares fitting being linear to the values of the data, the coefficients can be computed independent of data. The general coefficient matrix becomes  $C = (A^T A)^{-1} A^T$ .  $C$  can then be reassembled back into a traditional looking filter of size  $N \times N$ . In order to smooth the image the first coefficient is used, higher order coefficients are used to calculate derivatives. The advantage of the Savitzky-Golay filter has the ability to preserve higher moments in the data and thus reduce smoothing on peak heights. In more homogeneous areas the smoothing approaches an average filter.

## 3 Adaptive Smoothing

The algorithm for adaptive smoothing implemented in this paper is adapted from Chen [5]. The technique measures two types of discontinuities in the image, local and spatial. From both these measures a less ambiguous smoothing solution is found. In short, the local discontinuities indicate the detailed local structures while the contextual discontinuities show the important features.

In order to measure the local discontinuities for each pixel the average of the  $\nabla$ 's around the pixel in the horizontal, vertical and diagonal is calculated to be  $E_{xy}$ . In order to measure the contextual discontinuities, a spatial variance  $\sigma_{xy}^2(R)$  is employed in a square kernel  $N_{xy}(R)$ .

This value of sigma is then normalised to  $\tilde{\sigma}_{xy}^2$  between the minimum and maximum variance in the entire image. A transformation is then added into  $\tilde{\sigma}_{xy}^2$  to alleviate the influence of noise and trivial features. It is given a threshold value of  $\theta_\sigma = (0 \leq \theta_\sigma \leq 1)$  to limit the degree of contextual discontinuities.

Finally, the actual smoothing algorithm runs through the entire image updating each pixels intensity value  $I_{xy}^t$ , where  $t$  is the iteration value.

$$I_{xy}^{t+1} = I_{xy}^t + \eta_{xy} \frac{\sum_{(i,j) \in N_{xy}(1) \setminus \{(x,y)\}} \eta_{ij} \gamma_{ij}^t (I_{i,j}^t - I_{x,y}^t)}{\sum_{(i,j) \in N_{xy}(1) \setminus \{(x,y)\}} \eta_{ij} \gamma_{ij}^t} \quad (2)$$

where,

$$\eta_{ij} = \exp(-\alpha \Phi(\tilde{\sigma}_{ij}^2(R), \theta_\sigma)), \quad (3)$$

$$\gamma_{ij}^t = \exp(-E_{ij}^t/S) \quad (4)$$

The variables  $S$  and  $\alpha$  determine to what extent the local and contextual discontinuities should be preserved during smoothing. If there are a lot of contextual discontinuities in the image then the value of  $\eta_{ij}$  will have a large influence on the updated intensity value. On the other hand, if there are a lot of local discontinuities then both  $\gamma_{ij}$  and  $\eta_{ij}$  will have the overriding effect, as  $\eta_{ij}$  is used for gain control of the adaption.

## 4 Nonlinear Diffusion Filtering

The standard blurring operation involving Gaussian filtering attempts to remove the noise at the expense of poor edge preservation and is given as;

$$S_{x,y} = I_{x,y} \circ \text{Gauss}(x, y, \sigma) \quad (5)$$

where  $S$  is the filtered image,  $I$  is the input image,  $\circ$  implements the 2D convolution,  $\text{Gauss}()$  is the 2D Gaussian function where  $\sigma$  is the scale parameter. The smoothing becomes more pronounced for higher values of the scale parameter but we can notice a significant attenuation of the signal at image boundaries. This result is highly undesirable for many applications like image segmentation and edge tracking where a precise identification of object boundaries is required.

To alleviate the problems associated with the standard Gaussian smoothing technique, Perona and Malik [3] proposed an elegant smoothing scheme based on non-linear diffusion. In their formulation the blurring would be performed within homogeneous image regions with no interaction between adjacent or neighbouring regions that share a common border. The non-linear diffusion procedure can be written in terms of the derivative of the flux function,  $\phi(\nabla I) = \nabla I \cdot D(\|\nabla I\|)$ , where  $\phi$  is the flux function,  $I$  is the image and  $D$  is the diffusion function. This equation can be implemented in an iterative manner and the expression required to implement the non-linear diffusion is illustrated in Eq. 6.

$$I_{x,y}^{t+1} = I_{x,y}^t + \lambda \sum_{R=1}^4 [D(\nabla_R I) \nabla_R I]^t \quad (6)$$

where  $I^t$  represents the image at iteration  $t$ ,  $R$  defines the 4-connected neighbourhood,  $D$  is the diffusion function,  $\nabla$  is the gradient operator that has been implemented as the 4 connected nearest-neighbour differences and  $\lambda$  is a parameter that takes a values in the range  $0 < \lambda < 0.25$ .

The diffusion function  $D(x)$  should be bounded between 0 and 1 and should have the peak value when the input  $x$  is set to zero. This would translate with no smoothing around the region boundary where the gradient has high values. In practice, a large number of functions can be implemented to satisfy this requirement and in our implementation we were using the exponential function proposed by Perona and Malik [3],  $D(\|\nabla I\|) = e^{-\left(\frac{\|\nabla I\|}{k}\right)^2}$ , where  $k$  is the diffusion parameter. The parameter  $k$  selects the smoothness level and the smoothing effect is more noticeable for high values of  $k$ .

## 5 Anisotropic Gaussian Smoothing

An anisotropic filter based on the familiar Gaussian model was implemented in order to provide edge enhancing, directional smoothing. The goal was to develop a versatile smoothing filter based on a straightforward and highly adaptable form. The approach reduces to convolution with a scaled and shaped Gaussian mask, where the determination of the mask weights becomes the key step governing the performance of the filter. By calculating the local greyscale gradient vector and favouring smoothing along the edge over smoothing across it we can achieve an effective boundary preserving filtering approach, where regions are homogenized while edges are retained.

The weight  $wt(\vec{p}\vec{q}, \nabla u)$  at each location in the mask is a function of the local gradient vector at the centre of the mask and the distance of the current neighbour from that centre. There is a large number of possibilities for the formulation of the mask weight calculation, based on the desired form for the non-linear and anisotropic components of the filter. The weight for some neighbour  $q$  is calculated as a function of the gradient of point  $p$ , at the mask origin, and the distance from the origin to the neighbour  $q$ . The relationship used in our approach is given in Eq. 7, where  $\vec{p}\vec{q}$  is the vector from the mask centre point  $p$  to some neighbour  $q$ ,  $\nabla u$  is the gradient vector at  $p$ ,  $\lambda$  is the scale parameter, controlling smoothing strength, and  $\mu$  is the shape parameter, controlling anisotropy. When  $\mu$  equals zero the anisotropic term

$(\frac{\vec{p}\vec{q}\cdot\nabla u}{\lambda})^2(2\mu + \mu^2)$  disappears and the filter reduces to the non-linear, isotropic form, where smoothing decreases close to strong edges but is applied equally in all directions, at any given location in the image.

$$wt(\vec{p}\vec{q}, \nabla u) = e^{-((\frac{\|\vec{p}\vec{q}\| \|\nabla u\|}{\lambda})^2 + (\frac{\vec{p}\vec{q}\cdot\nabla u}{\lambda})^2(2\mu + \mu^2))} \quad (7)$$

The images in Figures 1 and 2 illustrate the operation of the anisotropic filter. As the smoothing strength and the number of iterations is increased more noise and small features are eliminated, but even in extreme cases the most important edges in the image are well preserved in both location and strength.

## 6 Experiments and Results

The results of each filter are assessed by their ability to smooth homogeneous areas while preserving the areas with higher moments. Smoothing of homogeneous areas is measured using the standard deviation while the preservation of edges is measured using the strength and spread of the edge in the filtered images. The filters are tested on two images, see figures 1 and 2, the first image of a laboratory having a high SNR (signal-noise-ratio) and high CNR (contrast-to-noise-ratio) with a high density of edges. The second medical image has a much lower SNR and CNR. Parameters were chosen to give the optimal results on visual inspection. Visual results are presented in figures 1 and 2. The standard deviation is



Figure 1: Results from each of the smoothing filters, top-row, l-to-r is the original image, image after Savitzky-Golay and Gaussian. Bottom-row, Adaptive, Nonlinear Diffusion and Anisotropic Gaussian.

measured in a  $7 \times 7$  window over the entire original image. From these values 25% of the highest values were eliminated as belonging to edges in the image and 25% of the lower values as having no significant texture to smooth. The standard deviation for each of the filtered images is then taken at the same positions. The results are presented in Table 1. For the laboratory image, Adaptive smoothing gives the best results followed by the two other non-linear filters. Both linear Savitzky-Golay and Gaussian filters have the highest deviation after smoothing. In the medical image there are more significant differences

	<i>Laboratory Image</i>			<i>MR Image</i>		
	SD	Edge height	Edge width	SD	Edge height	Edge width
Original	57.4	31	2.26	277.7	219	2.04
Savitzky-Golay	40.8	23	2.5	61.23	158	2.48
Gaussian	41.0	15	4.4	102.8	196	2.16
Adaptive	24.2	26	2.13	42.99	211	2.00
Diffusion	27.7	25	2.17	69.63	214	2.00
Anisotropic	31.9	30	2.17	35.05	219	1.99

Table 1: Shows the standard deviation (SD), edge strength and edge spread on both images after each filtering. The edge strength and edge spread are taken from histograms in figure 3.

with the anisotropic and adaptive smoothing operators giving the best results while the Gaussian performs worst in the low SNR image.

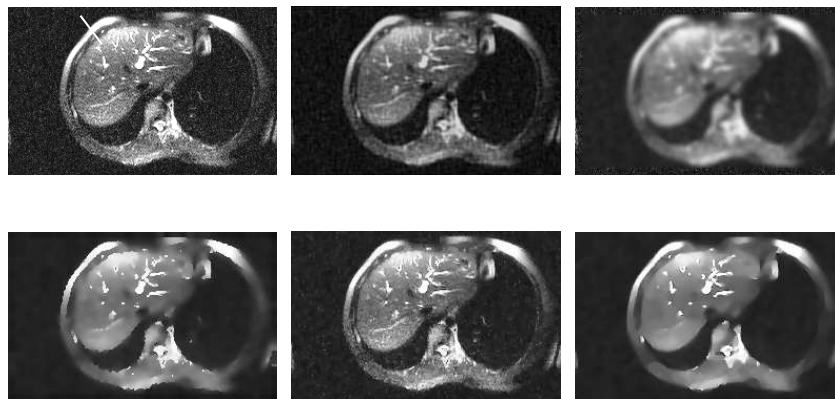


Figure 2: Results from each of the smoothing filters, top-row, l-to-r is the original image, image after Savitzky-Golay and Gaussian. Bottom-row, Adaptive, Nonlinear Diffusion and Anisotropic Gaussian.

The strength, shift and spread of the edge is evaluated on each of the images. Histogram plots across two edges are shown in figure 3 showing both the image pixels and the gradient across the edge. For the lab image the results are similar for all filters with more significant differences between filters in the medical image. Two measurements are taken from these histograms which indicate edge strength and spread. These results are compiled in Table 1. While Savitzky-Golay and Gaussian filters spread the edge, the other three maintain and even enhance the edge characteristics.

## 7 Conclusion

Five filters were evaluated using two criteria, texture smoothing and edge preservation. The filters consisted of two linear filters, two non-linear isotropic and one non-linear anisotropic. The filters were tested on two images with high and low SNR and the results show that, particularly in the low SNR case, the anisotropic and adaptive filters performs much better than the linear filters at smoothing out the noise in homogeneous areas while still maintaining the edge strengths with minimum blurring across the edge.

The Gaussian performs the worst of all the filters. The Savitzky-Golay deals better at preserving the edges but again suffers in the lower SNR image. The anisotropic and adaptive smoothings preservation of edges allows for more aggressive smoothing on homogeneous areas.

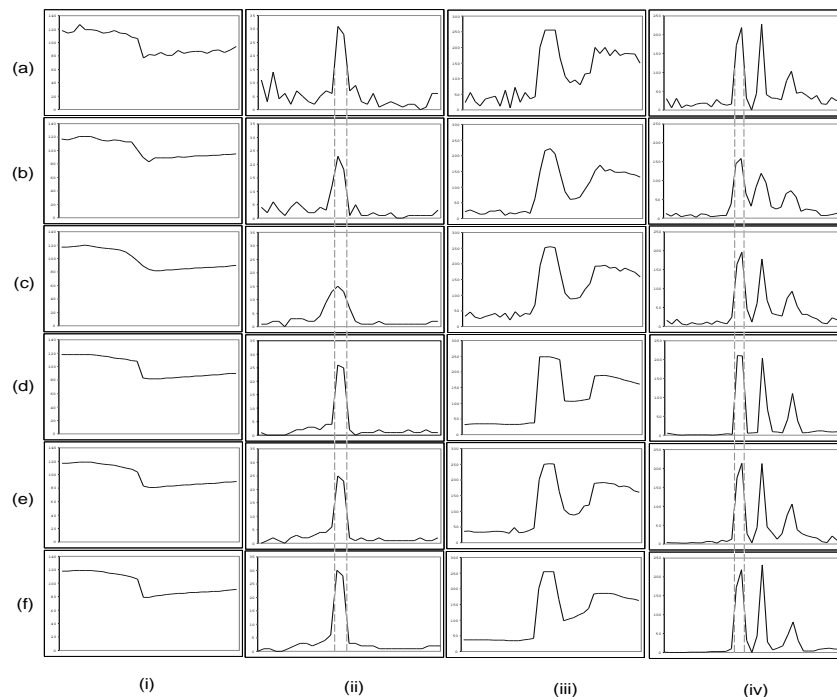


Figure 3: Pixel intensities and gradients along white lines from images *figure 1(a)* and *figure 2(a)*. (i) and (iii) show the pixel intensities and (ii) and (iv) show the gradient values from the lab image and the medical image respectively. (a) is the original image, (b) image after Savitzky-Golay, (c) Gaussian, (d) Adaptive, (e) Nonlinear Diffusion and (f) Anisotropic Gaussian.

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